Measuring the Spin parameter of the Massive black hole in the Galactic Center though star orbits

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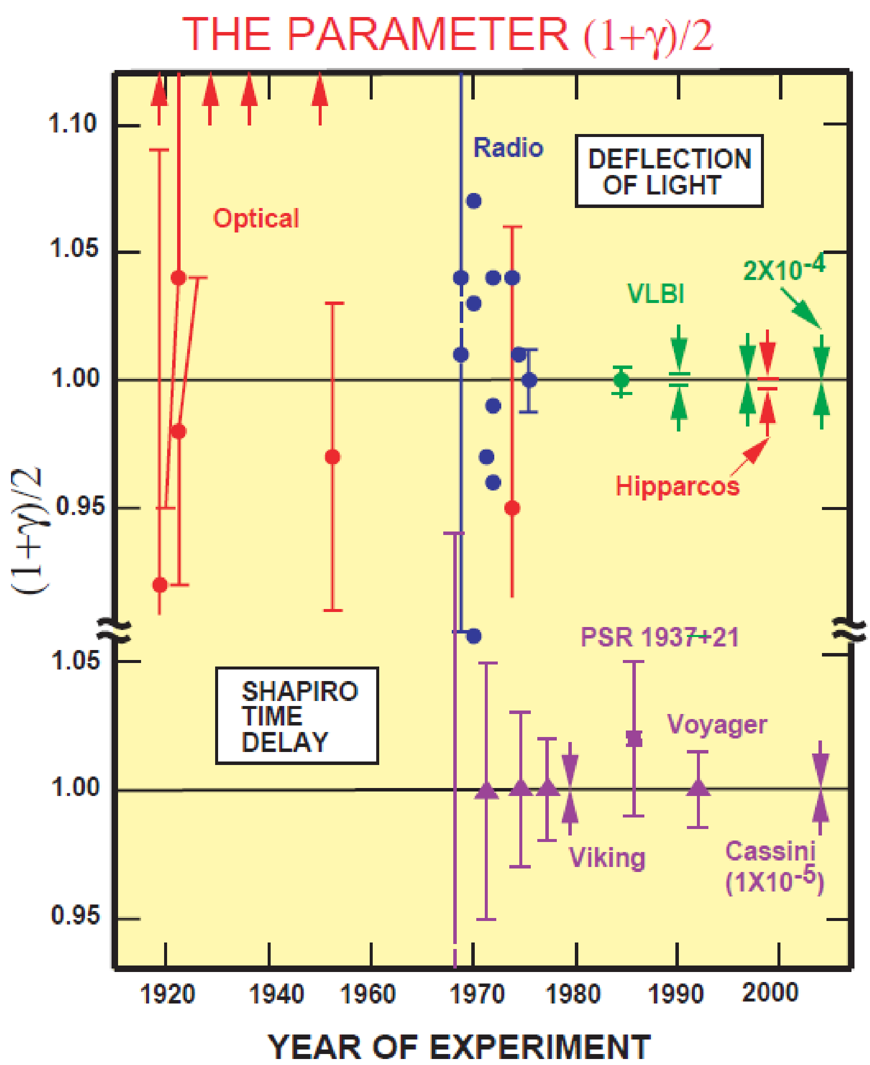
Theories of Gravity

- Alternative Gravity Theory
  - Tensor theories: f(R) gravity
  - Scalar-tensor theories: Brans-Dicke
  - Vector-tensor theories: Will-Nordtvedt

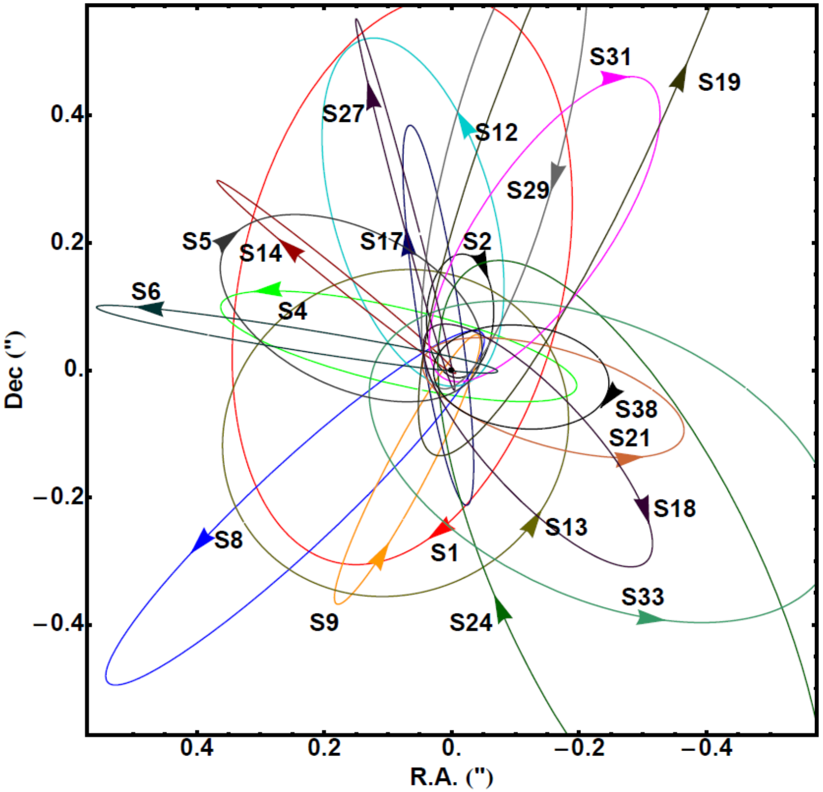
Parameter	Effect	Limit
$\gamma - 1$	time delay	2.3×10^{-5}
	light deflection	4×10^{-4}
$\beta - 1$	perihelion shift	3×10^{-3}
	Nordtvedt effect	2.3×10^{-4}
ξ	Earth tides	10^{-3}
α_1	orbital polarization	10^{-4}
		2×10^{-4}
α_2	spin precession	4×10^{-7}
α_3	pulsar acceleration	4×10^{-20}
η_N	Nordtvedt effect	9×10^{-4}
ζ_1	—	2×10^{-2}
ζ_2	binary acceleration	4×10^{-5}
ζ_3	Newton's 3rd law	10^{-8}
ζ_4	—	—



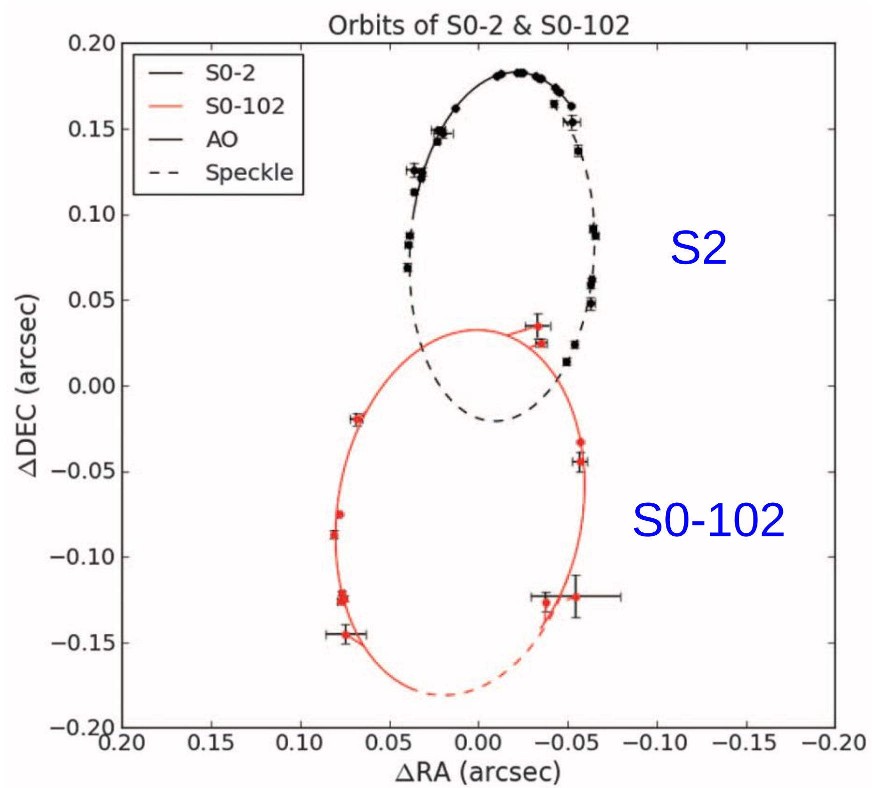
Clifford M. Will 2006

Strong field test of GR in the Galactic center

- GC MBH $4\times10^6\ m_{sun}$
- A clusters of young stars in the GC within 0.04pc
  - So called S-stars: 23, Randomized, eccentric orbits
  - Inner stars: S2(r_p= 3000r_g), S0-102

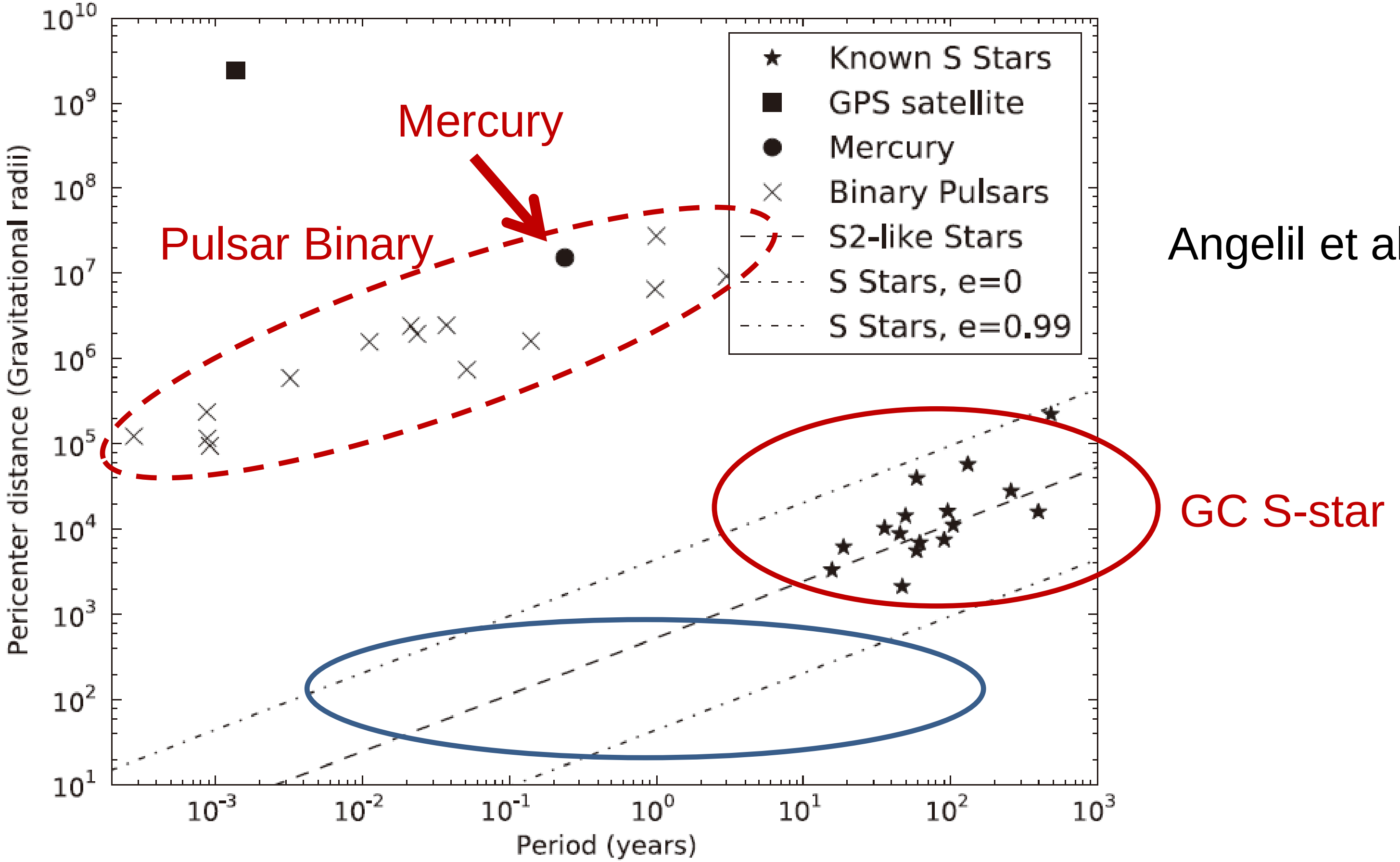


Gillessen et al. 2008



Meyer et al. 2012

Strong field test of GR in the Galactic center



Angelil et al. 2010

- Kerr metric is the most simple and elegant solutions to the Einstein field equation of GR

$$ds^2 = -(1 - 2Mr/\Sigma)dt^2 - (4Mar\sin^2\theta/\Sigma)dt d\varphi$$
$$+ (\Sigma/\Delta)dr^2 + \Sigma d\theta^2 + (r^2 + a^2 + 2Ma^2 r \sin^2\theta/\Sigma)\sin^2\theta d\varphi^2,$$

Inner S-stars

Two current detected stars

S2(980AU,0.88)

S0-102(850AU,0.68)

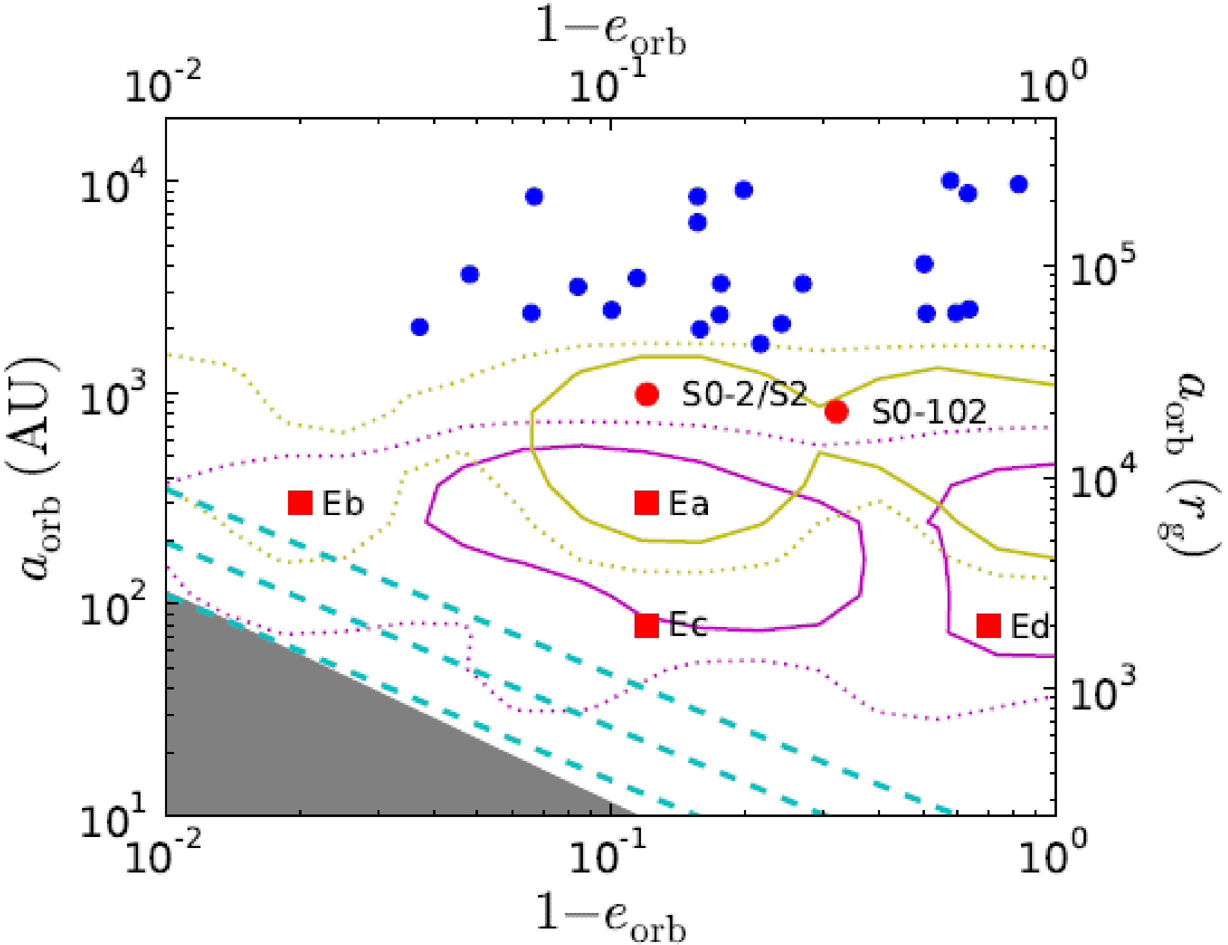
Other four inner stars

Ea (300AU, 0.88)

Eb (300AU, 0.98)

Ec (80AU, 0.88)

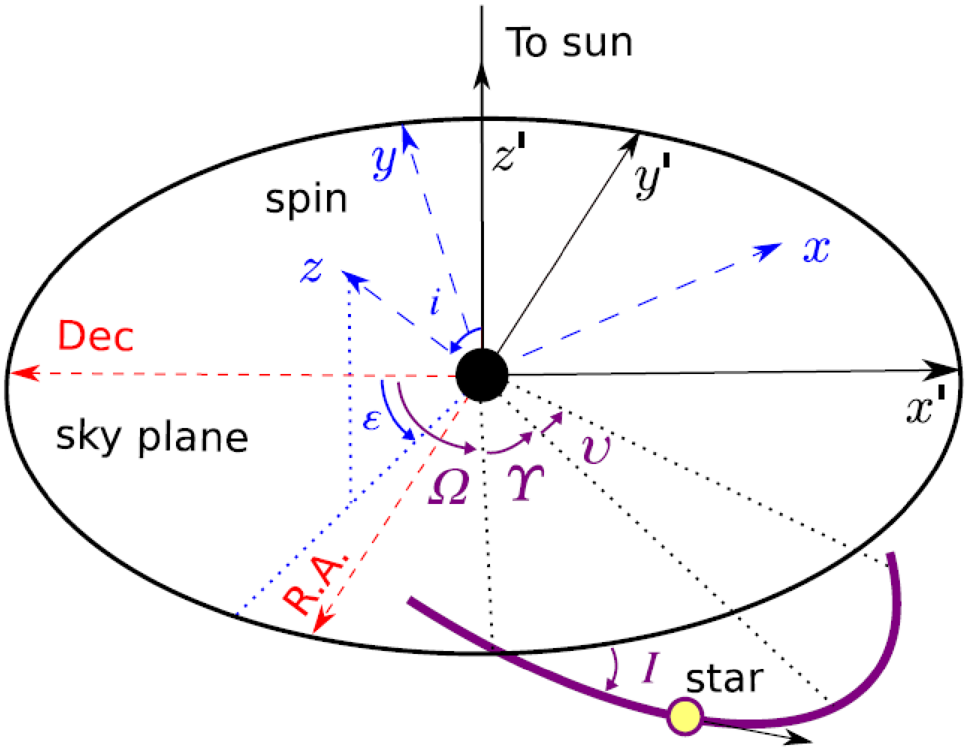
Ed (80AU, 0.3)



Full General Relativistic Simulation

- Motion Equations under Boyer-Lindquist Coordinate

$$\Sigma\dot{r} = \pm\sqrt{R},$$
$$\Sigma\dot{\theta} = \pm\sqrt{\Theta},$$
$$\Sigma\dot{\phi} = -a + \lambda/\sin^2\theta + aT/\Delta,$$
$$\Sigma\dot{t} = -a^2\sin^2\theta + a\lambda + (r^2+a^2)T/\Delta,$$



- Ray tracing Technique from Observer to the Star

$$\int_{r_0}^{r_{\rm hit}} \frac{dr}{\sqrt{R}} = \pm \int_{\mu_0}^{\mu_\star} \frac{d\mu}{\sqrt{\Theta_\mu}},$$

$$\phi = r_{\rm sign}\int^r \frac{\lambda r^2 + 2r(a-\lambda)}{r^2-2r+a^2}\frac{dr}{\sqrt{R(r)}} + \theta_{\rm sign}\int^\mu \frac{\lambda\mu^2}{1-\mu^2}\frac{d\mu}{\sqrt{\Theta_\mu}}$$

$$t = r_{\rm sign}\int^r \frac{r^4 + a^2r^2 + 2ar(a-\lambda)}{r^2-2r+a^2}\frac{dr}{\sqrt{R(r)}} + \theta_{\rm sign}\int^\mu a^2\mu^2\frac{d\mu}{\sqrt{\Theta_\mu}},$$

Full General Relativistic Simulation

- Red shift

$$z = \frac{\mathbf{p}_{\rm hit} \cdot \mathbf{u}_\star}{\mathbf{p}_{\rm o} \cdot \mathbf{u}_{\rm o}} - 1 = -\frac{\mathbf{p}_{\rm hit} \cdot \mathbf{u}_\star}{E_{\rm o}} - 1,$$

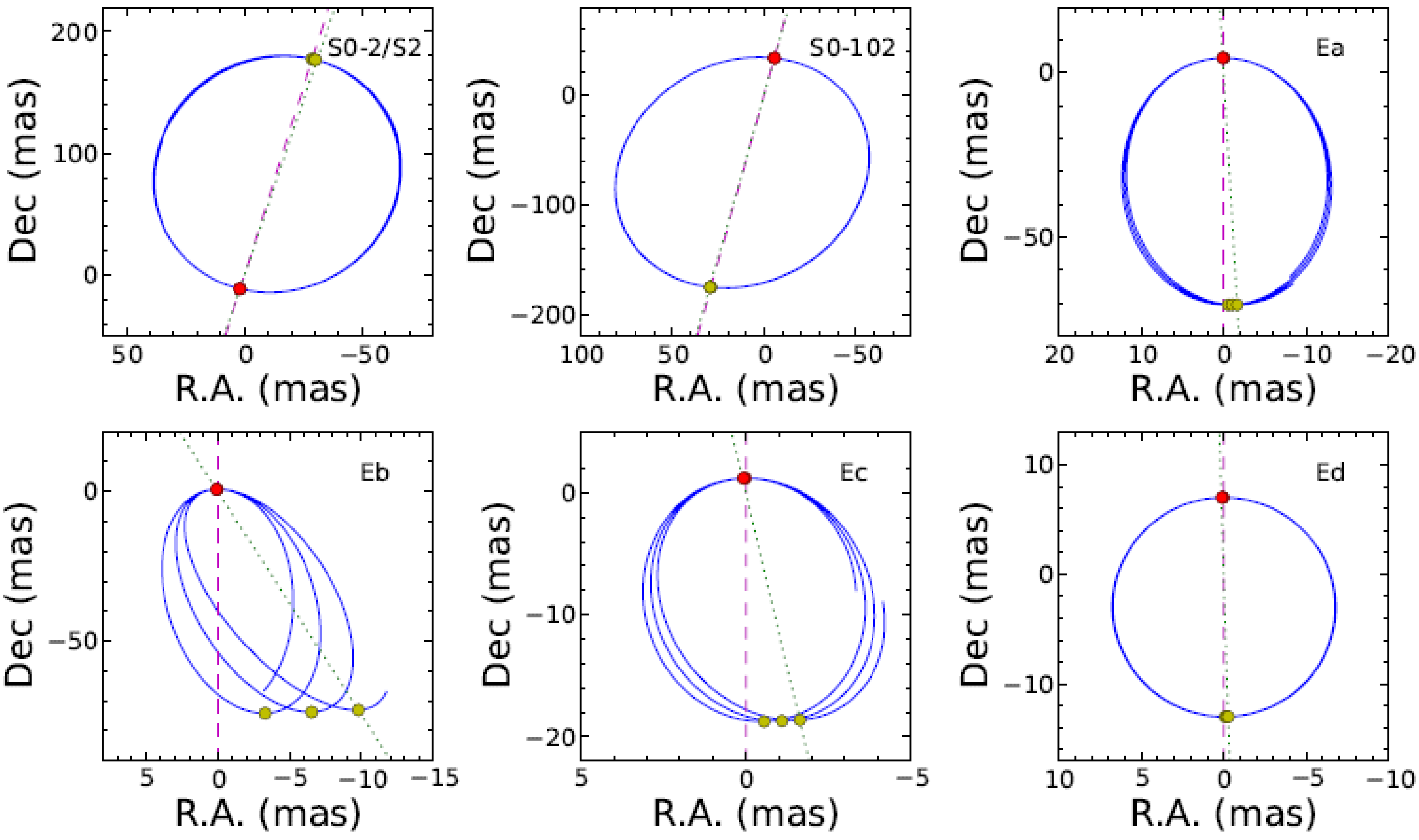
- Star image positions

$$\lambda = -\alpha \sin i,$$

$$q^2 = \beta^2 + (\alpha^2 - a^2) \cos^2 i.$$

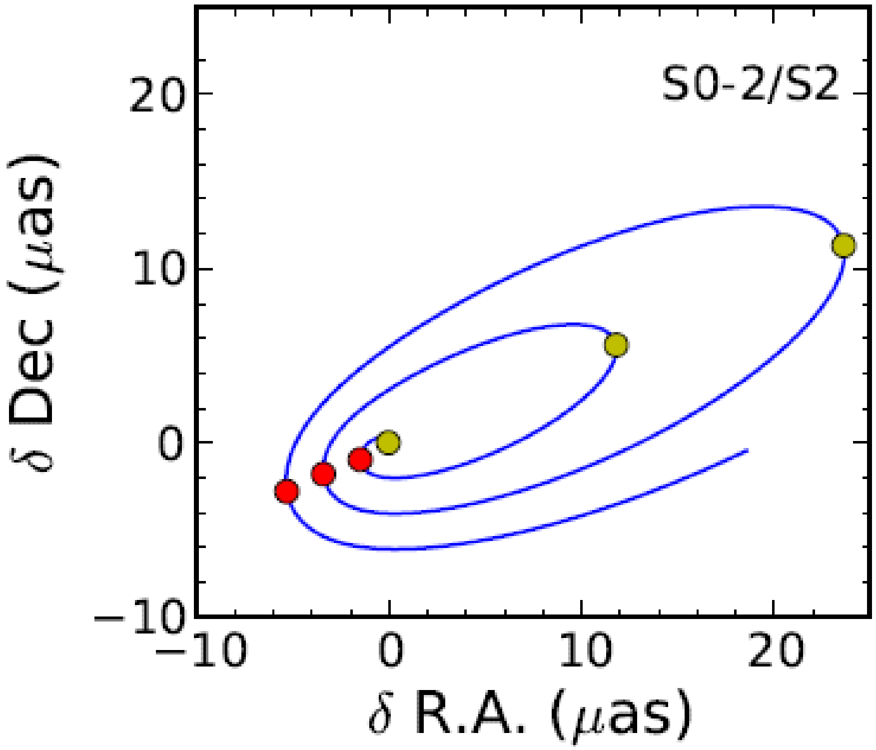
- Accuracies
    - redshift velocities: <~0.0001-0.001km/s
    - Images: < 0.01-0.001uas
- Fast -> MCMC

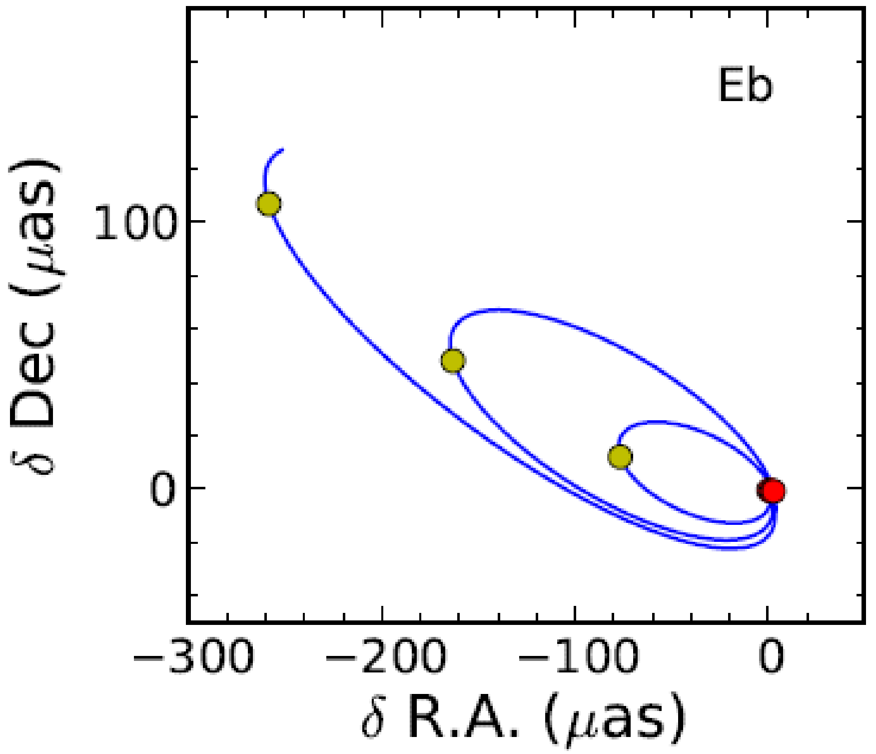
Trajectories of Stars in Three Period



Spin induced difference in star image position

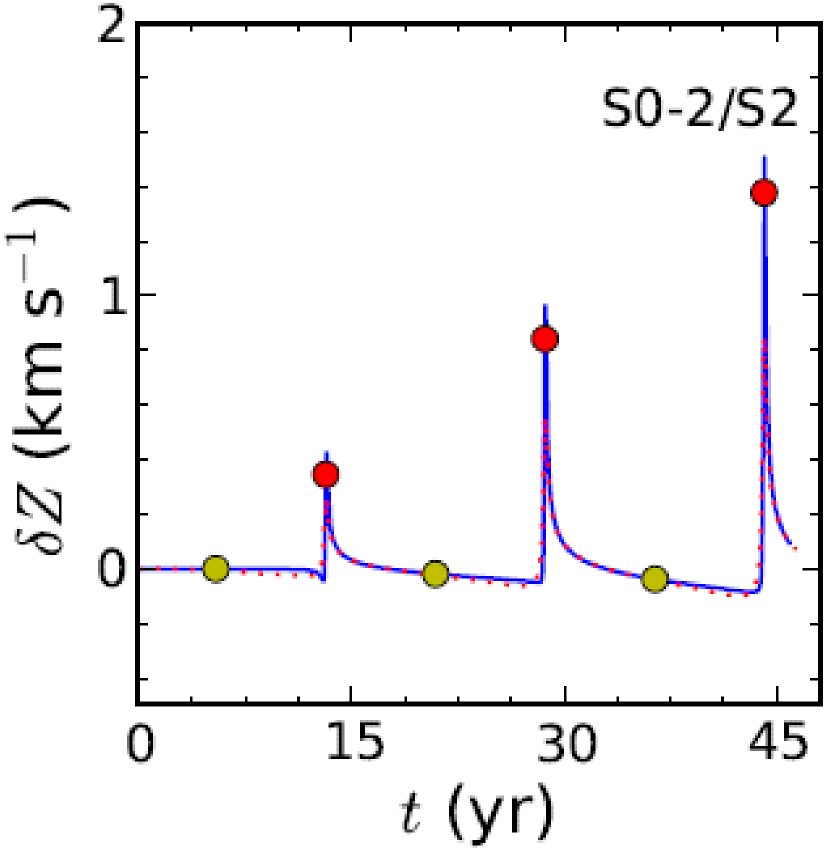
- Spiral like pattern
  - Different from perturbations

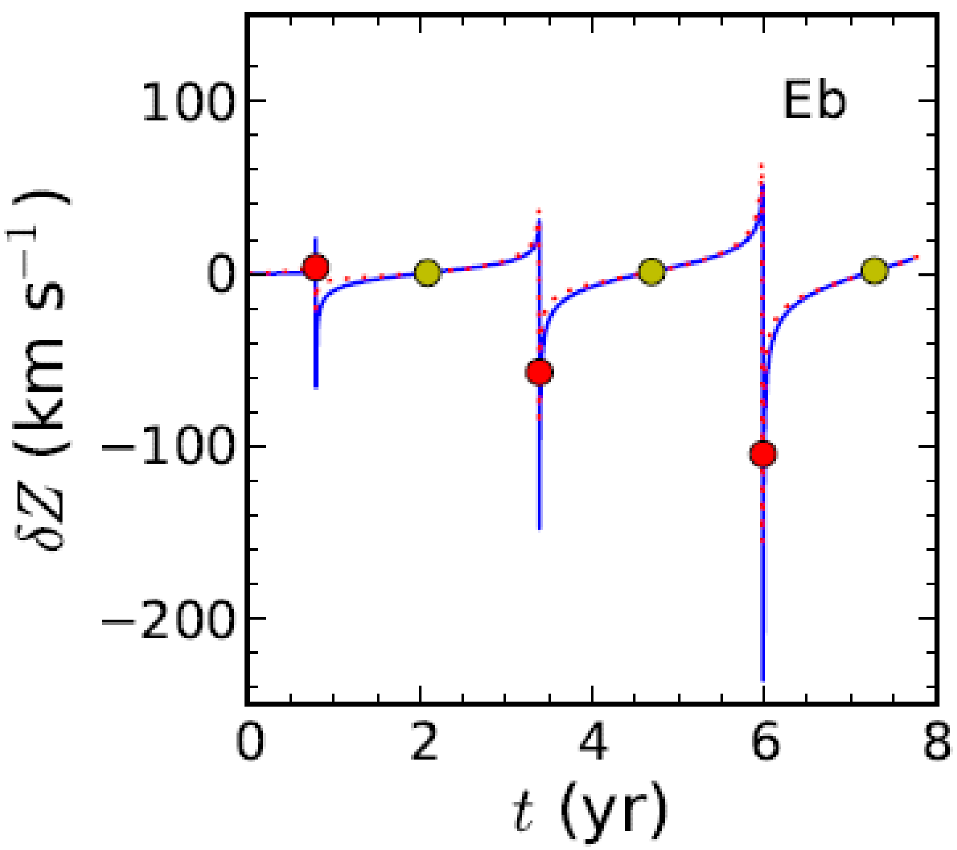




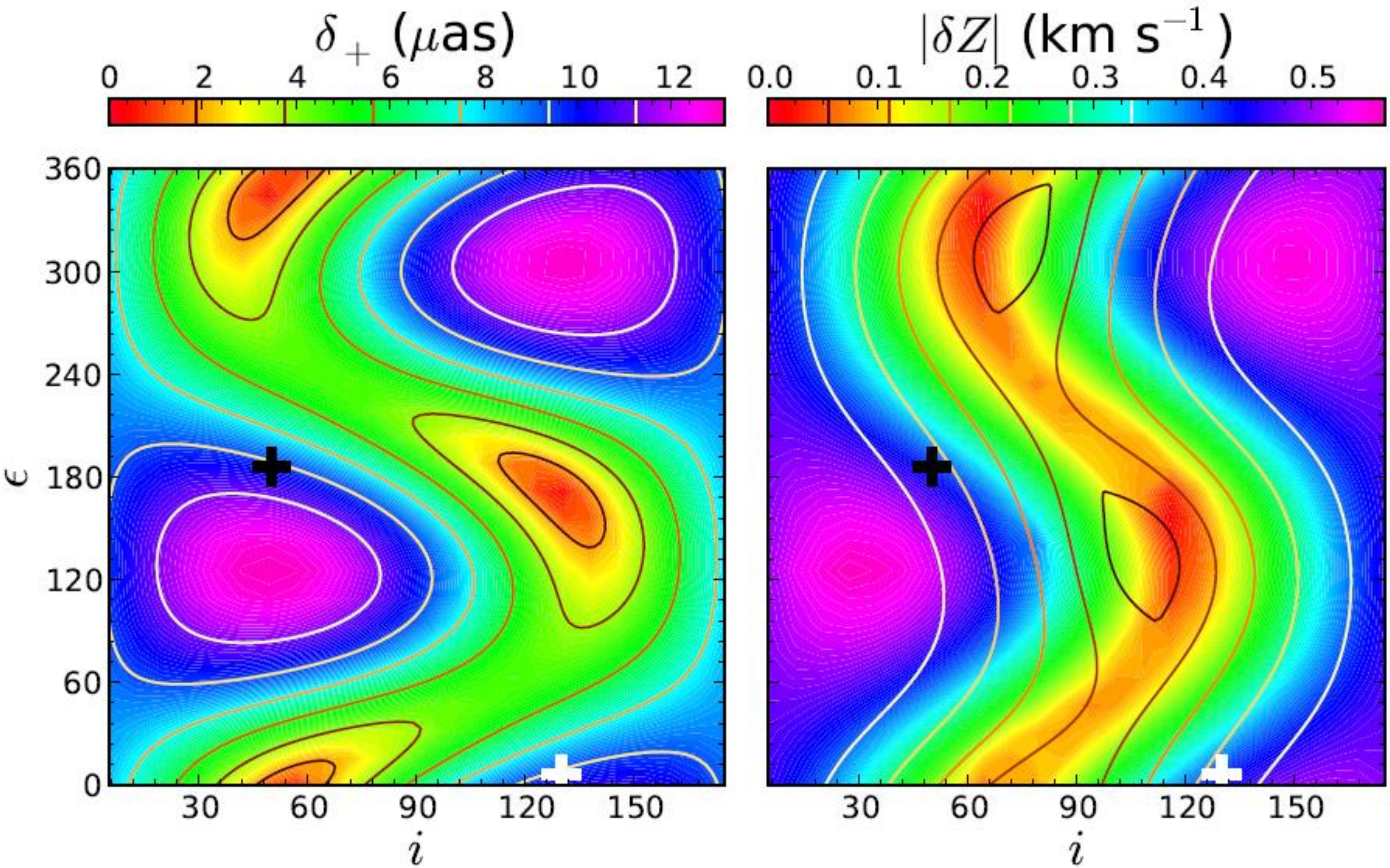
Spin induced difference in star redshift

- Velocity curve
  - Sharp at a specific position of the orbit

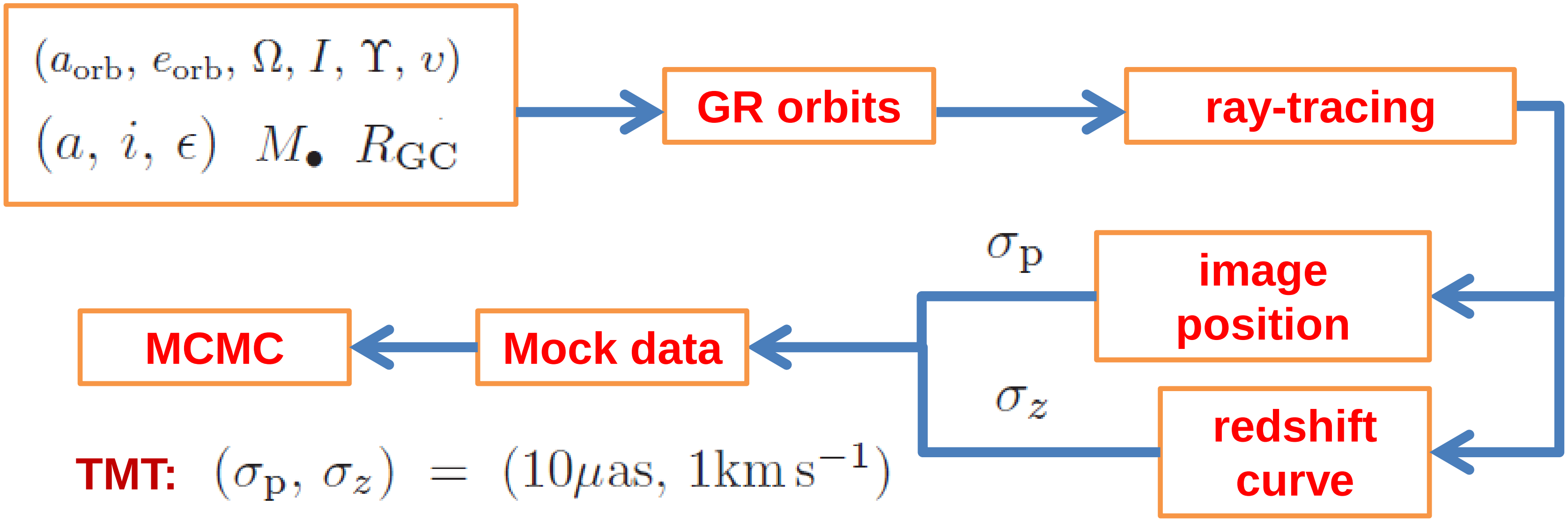




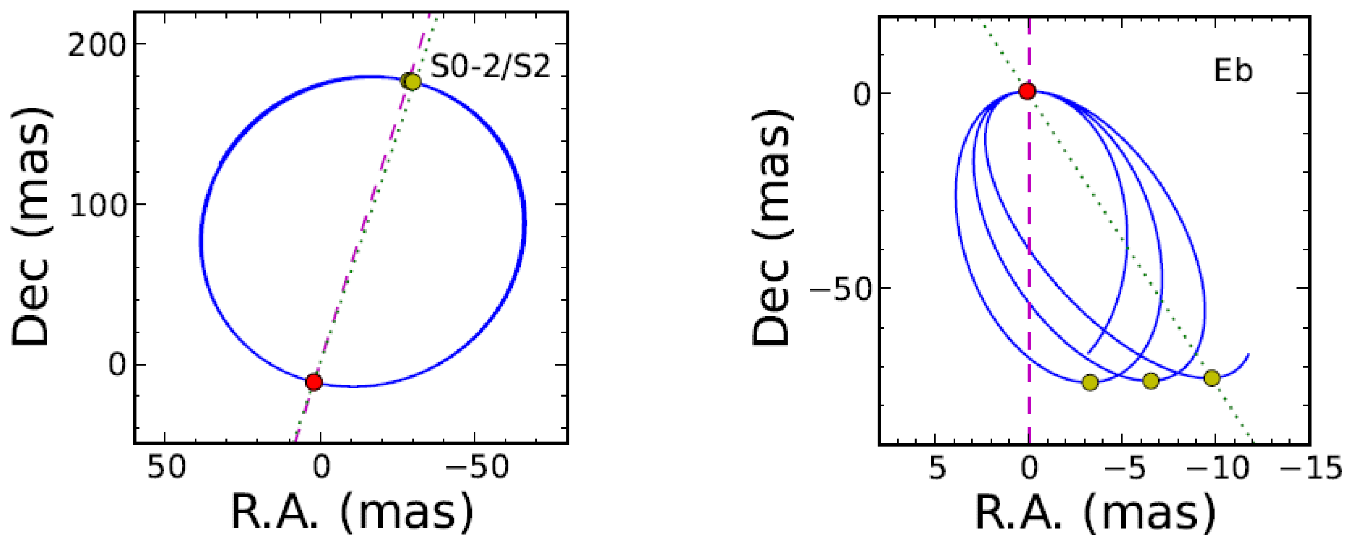
Dependence on the Spin direction



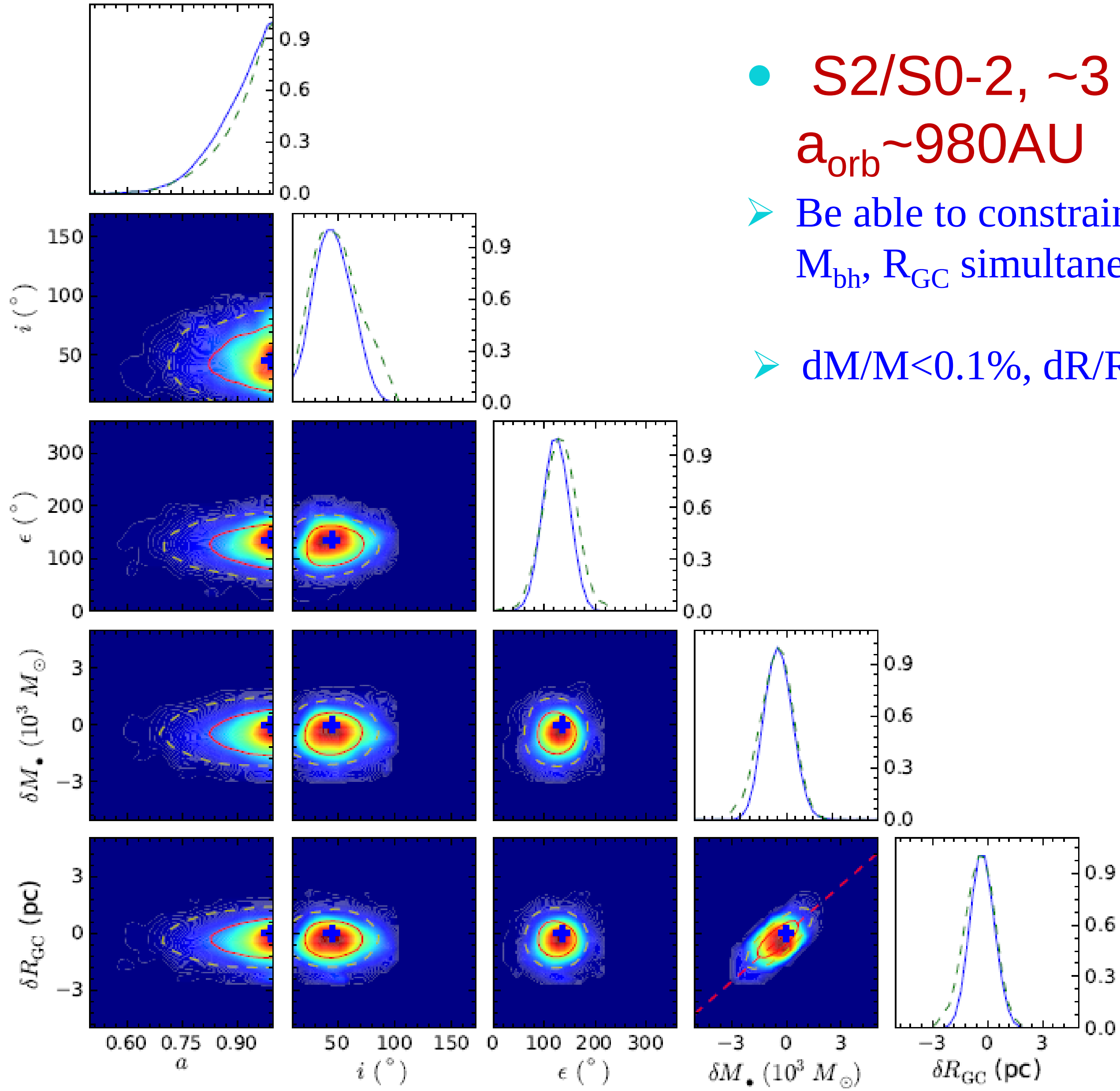
Markov Chain Monte Carlo Fitting



TMT: $(\sigma_{\rm p},\, \sigma_z) \;=\; (10\mu{\rm as},\, 1{\rm km\,s^{-1}})$



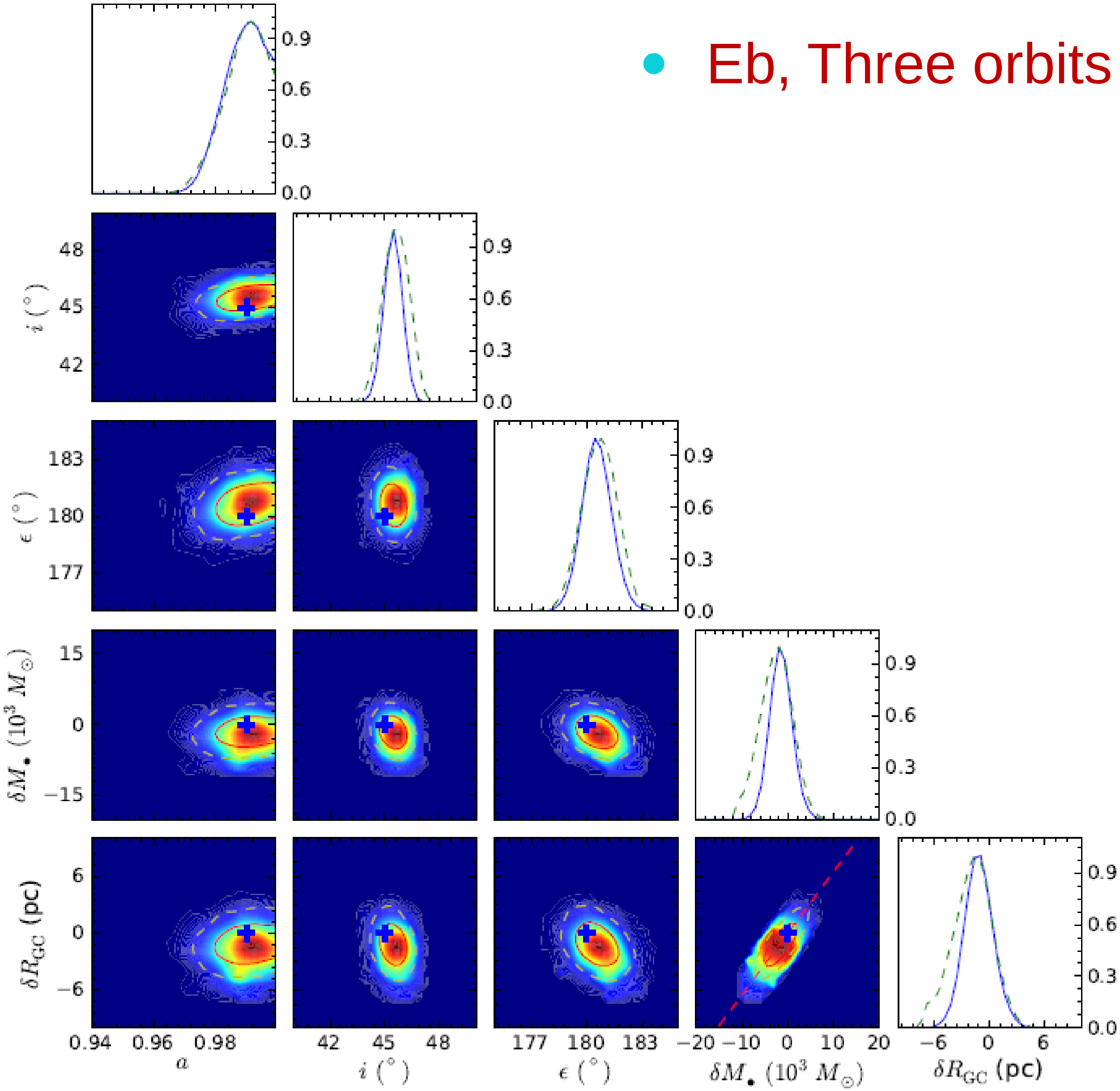
MCMC fitting of S2/S0-2



- S2/S0-2, ~3 orbits (30~45yr), a_{orb}~980AU
  - Be able to constrain spin, orientation of spin, M_{bh}, R_{GC} simultaneously
  - dM/M<0.1%, dR/R<0.01%

a	$0.812^{+0.187}_{-0.257}$
$\delta R_{\rm GC}$ (pc)	$-0.30^{+0.98}_{-0.97}$
$\delta M_\bullet$ $(10^3 M_\odot)$	$-0.41^{+1.17}_{-1.15}$

MCMC fitting of Eb



- Eb, Three orbits (~1 yr), a_{orb}=300AU

a	$0.989^{+0.010}_{-0.011}$
δR_{GC} (pc)	$-1.16^{+2.50}_{-2.34}$
$\delta M_\bullet$ ($10^3 M_\odot$)	$-1.53^{+3.55}_{-3.29}$

Summary

- The S-stars discovered around the massive black hole (MBH) in the Galactic Center, are anticipated to be able to provide unique dynamical constraint on the MBH spin parameter and the Kerr metric.
- We develop a **full GR framework** to simultaneously constrain the MBH mass, spin and orientations of spin by MCMC fitting techniques.
- We find that the spin-induced effects on the projected trajectory and redshift curve of a star can vary up to more than one order of magnitude for different spin directions.
- The S2/S0-2 can be used to constraining the MBH spin within 30~45yr by TMT or E-ELT. For more inner S-stars it will **take only ~1 yr**.
- The MBH mass and R_{GC} can be constrained to a **unprecedented accuracy (0.01%~0.1%)**
- Our result suggest that long term monitoring of the motions of stars in the GC by next generation telescopes can provide an ultimate dynamical test, **for the first time**, to the no-hair theorem and the gravity theory.

Thank you!~~